On the definitional character of axioms

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The search for new axioms for set theory

The main question:

What criteria should we follow when we evaluate new candidate axioms for set theory?

Self evidence, intrinsic motivations: "an axiom is a self-evident proposition requiring no formal demonstration to prove its truth, but received and assented as soon as mentioned" (Oxford dictionary). Some problems:

- Large cardinals axioms are not self-evident
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- even certain axioms of ZF strictly speaking are not obvious

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- Gödel: fruitfulness in terms of "verifiable consequences". Problem: as in the sense of experimental physics?
- Magidor: verifiable consequences are statements that were not refuted so far (ex. large cardinals imply PD that was not refuted so far). Problem: $V = L$ decides CH and has other consequences that were not refuted so far.
- Maddy: axioms are justified by the “proper methods” of the discipline (ex. PD arose naturally as a problem “proper” to descriptive set theory). Problem: what are exactly the proper methods? Isn’t CH a problem proper to set theory?

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A different approach: Axioms as definitions of a concept.

"*In my opinion, a concept can be fixed logically only by its relations to other concepts. These relations formulated in certain statements, I call axioms, thus arriving at the view that axioms (perhaps together with propositions assigning names to concepts) are the definitions of the concepts*". (Hilbert, letter to Frege 22 Sept. 1900)

Existential quantification in set theory

$$ZF \vdash \nexists y \forall x (x \in y)$$

If we read it as *"The class of all sets does not exist"*, then $\forall$ would refer only to 'sets' while $\exists$ would refer to actual existence.

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Rephrasing the axioms

Following our considerations we should rephrase the axioms roughly as follows:

- Axiom of Pairing: "given two sets x, y, the collection containing exactly x and y **is a set**"
- Axiom of Choice: "given a family of nonempty sets (the family itself is a set) the image of the choice function **is a set**"
- Axiom of Infinity: "the collection of sets $\mathcal{C}$ inductively defined by $x \in \mathcal{C} \to x \cup \{x\} \in \mathcal{C}$ **is a set**"
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Existential quantification in set theory- continue

- The axioms of *ZFC* do not establish the actual existence of something. Instead, they are ways of singling out things which are already taken to exist or are considered to be legitimate objects.
- the axioms *select* among such things, those that can have the title of ‘set’. This is the real meaning of existential quantification in set theory.

Indeed, in Zermelo’s set theory a domain of individuals was given, that included also ‘urelements’, namely objects that are not sets, but may be contained in some sets as elements. The axioms selected the ‘sets’ among the objects in the domain.

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Let us call $\mathfrak{U}$ this domain of legitimate objects.

The nature of the legitimate objects in $\mathfrak{U}$, i.e. whether they are names, fictions or at the opposite abstract entities platonically intended is irrelevant to evaluate the axioms of set theory. The axioms of set theory concern the predicate 'being a set'.

The Axiom of Choice

For certain axioms such as Choice, Infinity, Large Cardinals etc., the disagreement comes already with the assumption that the corresponding objects are legitimate, *before* the fact that these are ‘sets’ or not. In other words, the problem is with the assumption that they are in $\mathfrak{U}$.

- the fact that the choice function was used implicitly in many areas of mathematics without raising any objections, suggests that in general mathematicians consider the image of the choice function as a legitimate object for $\mathfrak{U}$
- although some may consider that legitimate objects are only finite objects, or constructive objects... for those persons the image of the choice function is not in $\mathfrak{U}$
- Banach-Tarski paradox follows from the attempt to perform certain **operations** with the image of the choice functions, operations that are definable from the other axioms of ZF. Thus it is only related to the claim that the image of the choice function *is a set*, not to the fact that the choice function is a legitimate notion.

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What does it mean to be a 'set'?

What makes a collection worth the title of 'set' is the possibility of performing certain operations on this collection.

- Example 1: the collection of all sets that do not belong to them selfs may very well be in $\mathfrak{U}$, we may very well claim that it exists platonically, or we can claim that it is logically well defined as it is not a source of antinomies *per se*. We have a paradox only if we attempt to predicate membership of this collection.
- Example 2: the class of all sets may be in $\mathfrak{U}$, it is also not problematic *per se*. We have a contradiction only if we make it available for the other operations definable in ZF: let $\mathcal{C}$ be the collection of all sets, then apply the axiom of comprehension to say that the collection $\mathcal{C}'$ of all *sets* that do not belong to them selfs *is a set*, then ask whether or not $\mathcal{C}'$ belongs to it self.

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Axioms as definitions

When we infer '$\exists$' of a certain set, we make this object available for the operations definable from the other axioms. Thus being a set means being available for the other axioms. It follows that the meaning of 'set' depends on the theory. In this sense, the theory ZF (or ZFC) as a whole is a **definition** of 'set'.

"*[...] to give a definition of a point in three lines is to my mind an impossibility, for only the whole structure of axioms yields a complete definition and hence every new axioms changes the concept.*"(Hilbert 29 Dic. 1899)

The Frege-Hilbert controversy

" *[...] it is surely obvious that every theory is only a scaffolding or schema of concepts together with their necessary relations to one another, and that the basic elements can be thought in any way one likes. If in speaking of my points I think of some system of things, e.g. the system: love, law, chimney-sweep... and then assume all my axioms as relations between these things, then my propositions, e.g. Pythagoras' theorem, are also valid for these things.* "
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In our terminology, the domain $\mathfrak{U}$ of legitimate objects may change, the theory is about the schema of concept of set.

The Frege-Hilbert controversy

"*At the same time, the further a theory has been developed and the more finely articulated its structure, the more obvious the kind of application it has in the world of appearances and it takes a very large amount of ill will to want to apply the more subtle propositions of plane geometry or of Maxwell's theory of electricity to other appearances than the ones for which they were meant...*"
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Axioms as definitions

- The meaning of ‘set’ changes if we change the theory. So being a set in the sense of ZF is different than in ZFC, or ZFC+V=L etc.
- This does not imply that mathematics consists of random definitions with their logical consequences. We may have an innate concept of ‘set’, or the concept of ‘set’ may be somehow ‘real’, then one theory of sets can be ‘truer’ than another if the concept of ‘set’ defined is the intended one.

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Is there a universal, intersubjective concept of sets?

Maybe not, because the discussion around different theories such as V=L, V=Ultimate L, Forcing axioms etc. brings out different intuitions of the concept of set.

Following our terminology, V is the result of our selection, namely the objects in $\mathfrak{U}$ that have the property of being 'sets'.

- V is constructed by a sequence of stages where each stage is obtained from the previous stage by an operation which is definable in a "canonical" way...
  - ... by taking the next stage as the definable subsets of the previous stage (V=L)
  - ... at each stage you throw in the minimal (partial) extender missing on the sequence constructed thus far, to get an inner models of some large cardinal (this leads us to V= Ultimate L)
- anything that can be forced by some "nice" forcing is in V (Forcing axiom)

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The continuum hypothesis is an inherently vague question

The meaning of $\mathfrak{c}$ depends on the theory. We may agree on what is $\mathbb{N}$ and the parts of $\mathbb{N}$, but which of those parts are "sets" depends on the meaning of "set", that depends on the theory.

Example: in the framework of Forcing Axioms, all the parts that can be forced are "sets", thus $\mathfrak{c}$ is quite large, hence CH fails ($\mathfrak{c} = \aleph_2$). In the framework of V=L (or even V=Ultimate L) the notion of 'set' is more restrictive thus only few parts are 'sets', hence CH holds.

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Set theory, a theory like any other

If the axioms of set theory are definitions of a concept of set, then they are not substantially different than the axioms that define the notion of group, ring etc.

However...

The concept of set is supposed to be rich enough to embrace all the standard mathematical notions, including groups, rings etc.

This is the *foundational role* of set theory.

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Some concepts of sets may be more "expressive" than others: you can express 'set in the sense of V=L' in the theory of large cardinals (by simply saying that the object in question is in L), the converse is not true.

(Steel) "The language of set theory as used by the believer in V = L can certainly be translated into the language of set theory as used by the believer in measurable cardinals, via the translation $\varphi \mapsto \varphi^L$. There is no translation in the other direction."

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- when we formulate a set theory, we assume a universe of legitimate objects $\mathfrak{U}$ and the theory selects which of those objects have the property of being ‘sets’
- there is a multitude of legitimate concepts of sets, maybe there is not a universal one
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