Reversible Barbed Congruence on Configuration Structures

Groupe de Travail Analyse et Conception de Systèmes

Clément Aubert[1,3] Ioana Cristescu[2,3]

[1]LACL - Univ. Paris Est Creteil

[2]PPS - Univ. Paris Diderot

[3]ANR-14-CE25-0005 ELICA & ANR-11-INSE-0007 REVER

28 mai 2015

$$P = (a.b.0) + (b.a.0)$$

$$Q = (a.0)|(b.0)$$

$$P = (a.b.0) + (b.a.0) \xrightarrow{a} b.0$$

$$Q = (a.0)|(b.0)$$

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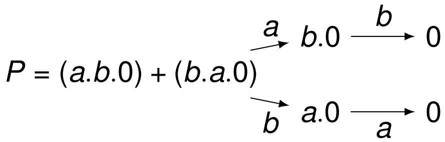
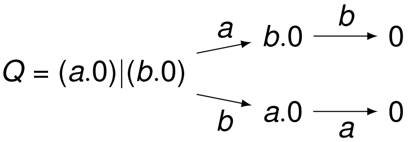
$$Q = (a.0)|(b.0) \xrightarrow{a} 0|(b.0)$$

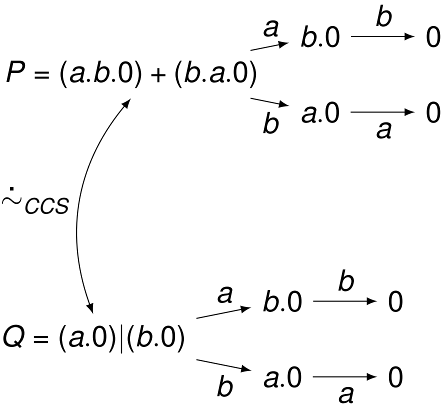
$$Q = (a.0)|(b.0) \xrightarrow{b} (a.0)|0$$

$$P = (a.b.0) + (b.a.0) \begin{array}{l} \xrightarrow{a} b.0 \xrightarrow{b} 0 \\ \xrightarrow{b} a.0 \xrightarrow{a} 0 \end{array}$$

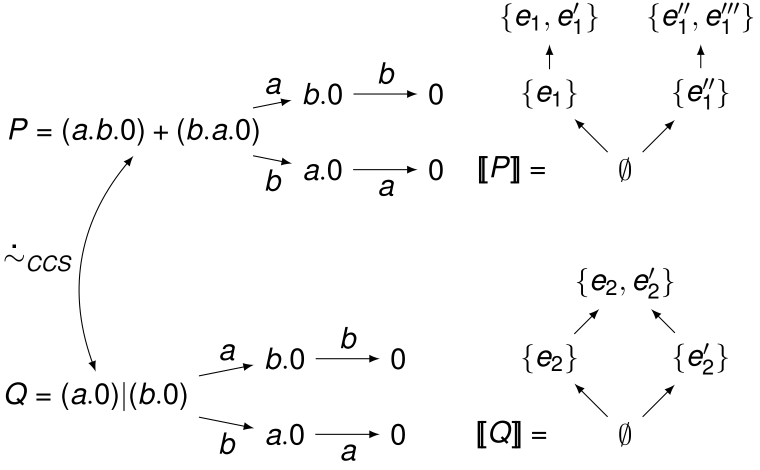
$$Q = (a.0)|(b.0) \begin{array}{l} \xrightarrow{a} b.0 \\ \xrightarrow{b} a.0 \end{array}$$


P = (a.b.0) + (b.a.0)
a
b.0
b
0
b
a.0
a
0


Q = (a.0)|(b.0)
a
b.0
b
0
b
a.0
a
0

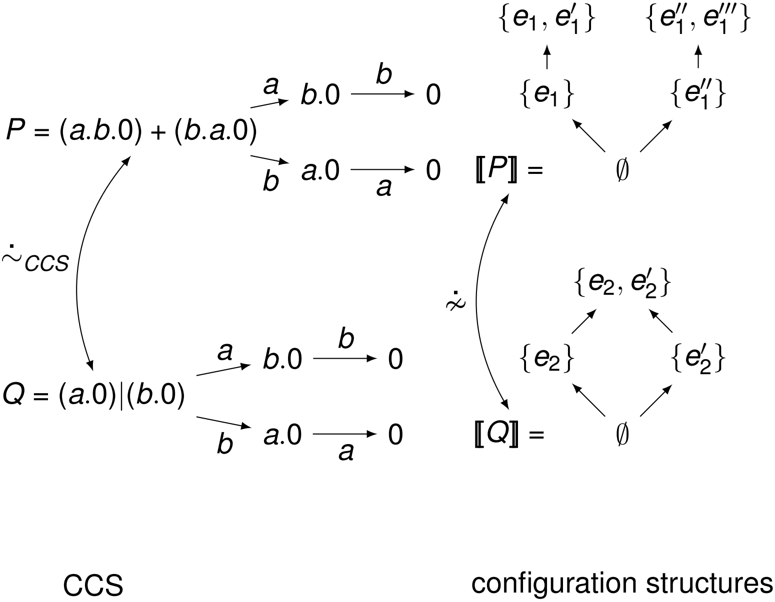


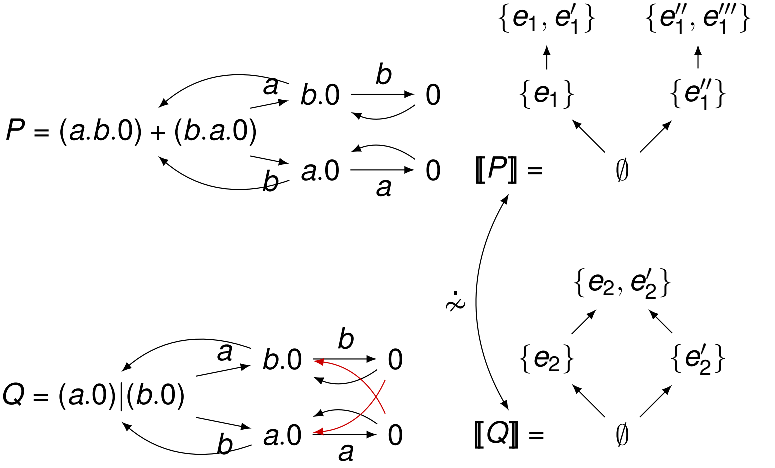
CCS



CCS

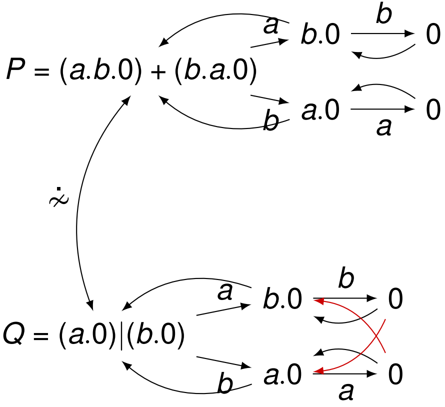
configuration structures

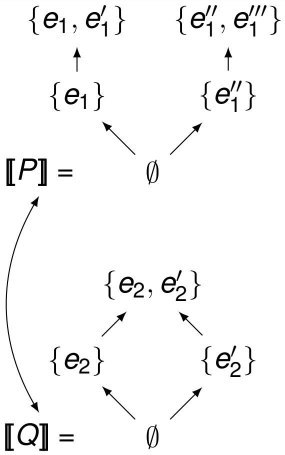

$P = (a.b.0) + (b.a.0)$
a
$b.0$
b
0
b
$a.0$
a
0
$\dot\sim_{CCS}$
$Q = (a.0)|(b.0)$
a
$b.0$
b
0
b
$a.0$
a
0
$[\![P]\!] =$
$\emptyset$
$\{e_1\}$
$\{e_1''\}$
$\{e_1, e_1'\}$
$\{e_1'', e_1'''\}$
$\dot\nsim$
$[\![Q]\!] =$
$\emptyset$
$\{e_2\}$
$\{e_2'\}$
$\{e_2, e_2'\}$
CCS
configuration structures



RCCS

configuration structures





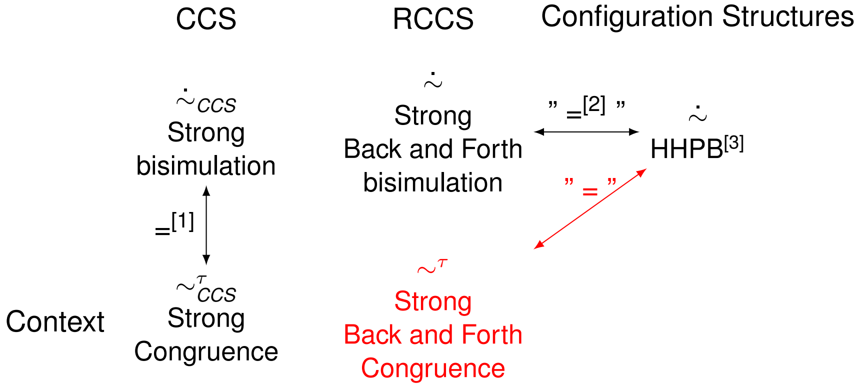
RCCS

configuration structures

CCS: *labelled* equivalences $\Leftrightarrow$ contextual

$P \sim^{\tau} Q \Leftrightarrow$ if for every *context* C, $C[P]$ and $C[Q]$ have the same *observables*.

contextual characterisation of
hereditary history-preserving bisimulation (hhpb)?



[1] Milner and Sangiorgi (1992). "Barbed Bisimulation"
[2] Phillips and Ulidowski (2012). "Reversibility and Models for Concurrency"
[3] Bednarczyk (1991). "Hereditary history preserving bisimulation"

$$\alpha, \beta := a \ \| \ \overline{a} \ \| \ \ldots \ \| \ \tau \qquad \text{(Actions)}$$

$$P, Q := 0 \ \| \ \alpha.P \ \| \ \alpha.P + \beta.Q \ \| \ P|Q \qquad \text{(CCS processes)}$$

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \ldots \;\|\; \tau \qquad \text{(Actions)}$$

$$P, Q := 0 \;\|\; \alpha.P \;\|\; \alpha.P + \beta.Q \;\|\; P|Q \qquad \text{(CCS processes)}$$

$$\ldots \qquad \frac{}{a.P \xrightarrow{a} P} \qquad \frac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \text{ syn.} \qquad \ldots$$

Calculus of Communicating Systems (CCS) in one slide

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \dots \;\|\; \tau \qquad \text{(Actions)}$$

$$P, Q := 0 \;\|\; \alpha.P \;\|\; \alpha.P + \beta.Q \;\|\; P|Q \qquad \text{(CCS processes)}$$

$$\dots \qquad \frac{}{a.P \xrightarrow{a} P} \qquad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \text{ syn.} \qquad \dots$$

Contextual equivalence

$P \sim^{\tau} Q \Leftrightarrow$ if for every *context* C, $C[P]$ and $C[Q]$ have the same *observables*.

Calculus of Communicating Systems (CCS) in one slide

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \dots \;\|\; \tau \qquad \text{(Actions)}$$

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Contextual equivalence

$P \sim^{\tau} Q \Leftrightarrow$ if for every *context* C, $C[P]$ and $C[Q]$ have the same *observables*.

Context

$C := [\,] \;\|\; \alpha.C \;\|\; C + P \;\|\; C|P$.

Observable (barb)

$P \downarrow_{\alpha}$ if there exists P' such that $P \xrightarrow{\alpha} P'$.

$$\alpha, \beta := a \;\|\; \overline{a} \;\|\; \ldots \;\|\; \tau \qquad \text{(Actions)}$$

$$P, Q := 0 \;\|\; \alpha.P \;\|\; \alpha.P + \beta.Q \;\|\; P|Q \qquad \text{(CCS processes)}$$

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Strong barbed congruence

$$P \overset{\cdot}{\sim}^{\tau}{}_{CCS}\, Q \Leftrightarrow \begin{cases} Q \overset{\cdot}{\sim}^{\tau}{}_{CCS}\, P \\ P \xrightarrow{\tau} P' \Rightarrow Q \xrightarrow{\tau} Q' \wedge P' \overset{\cdot}{\sim}^{\tau}{}_{CCS}\, Q' \\ P \downarrow_{\alpha} \Rightarrow Q \downarrow_{\alpha} \end{cases}$$

$$P \sim^{\tau}{}_{CCS}\, Q \Leftrightarrow \forall C, C[P] \overset{\cdot}{\sim}^{\tau}{}_{CCS}\, C[Q]$$

$$R, S := m \rhd P \;\|\; R|R \qquad \text{(RCCS processes)}$$

$$m := \emptyset \;\|\; \curlyvee.m \;\|\; \langle i, a, P\rangle.m \;\|\; \langle i, a\rangle.m \qquad \text{(Memories)}$$

$$R, S := m \rhd P \;\parallel\; R|R \qquad \text{(RCCS processes)}$$
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Example

$$\emptyset \rhd (a.P + b.Q)|(c.\overline{a}.P')$$

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$$\equiv (\curlyvee.\emptyset \rhd (a.P + b.Q))|(\curlyvee.\emptyset \rhd (c.\overline{a}.P'))$$

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$$\equiv (\curlyvee.\emptyset \rhd (a.P + b.Q))|(\curlyvee.\emptyset \rhd (c.\overline{a}.P'))$$

$$\xrightarrow{1:b}(\langle 1, b, a.P\rangle.\curlyvee.\emptyset \rhd Q)|(\curlyvee.\emptyset \rhd (c.\overline{a}.P'))$$

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$$\equiv (\curlyvee.\emptyset \triangleright (a.P + b.Q))|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$
$$\xrightarrow{1:b} (\langle 1, b, a.P\rangle.\curlyvee.\emptyset \triangleright Q)|(\curlyvee.\emptyset \triangleright (c.\overline{a}.P'))$$
$$\xrightarrow{2:c} (\langle 1, b, a.P\rangle.\curlyvee.\emptyset \triangleright Q)|(\langle 2, c\rangle.\curlyvee.\emptyset \triangleright (\overline{a}.P'))$$

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$$\rightsquigarrow^{1:b} (\curlyvee.\emptyset \triangleright (a.P + b.Q))|(\langle 2, c\rangle.\curlyvee.\emptyset \triangleright (\overline{a}.P'))$$
$$\xrightarrow{3:\tau} (\langle 3, a, b.Q\rangle.\curlyvee.\emptyset \triangleright P)|(\langle 3, \overline{a}\rangle.\langle 2, c\rangle.\curlyvee.\emptyset \triangleright P')$$

$$R, S := m \rhd P \;\|\; R|R \qquad \text{(RCCS processes)}$$

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Strong back-and-forth barbed bisimulation

$$R \dot{\sim}^{\tau} S \Leftrightarrow \begin{cases} S \dot{\sim}^{\tau} R \\ R \rightarrow^{\tau} R' \Rightarrow S \rightarrow^{\tau} S' \wedge R' \dot{\sim}^{\tau} S' \\ R \rightsquigarrow^{\tau} R' \Rightarrow S \rightsquigarrow^{\tau} S' \wedge R' \dot{\sim}^{\tau} S' \\ R \downarrow_{\alpha} \Rightarrow S \downarrow_{\alpha} \end{cases}$$

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Strong back-and-forth barbed bisimulation

$$R \,\dot\sim^\tau\, S \Leftrightarrow \begin{cases} S \,\dot\sim^\tau\, R \\ R \rightarrow^\tau R' \Rightarrow S \rightarrow^\tau S' \wedge R' \,\dot\sim^\tau\, S' \\ R \rightsquigarrow^\tau R' \Rightarrow S \rightsquigarrow^\tau S' \wedge R' \,\dot\sim^\tau\, S' \\ R \downarrow_\alpha \Rightarrow S \downarrow_\alpha \end{cases}$$

Strong back-and-forth barbed congruence

$\sim^\tau = \dot\sim^\tau$ closed by *context*

Origin of a process

$$O_R = P \text{ such that } \emptyset \rhd P \rightarrow^* R$$

Simplest case: contexts and processes with an empty memory.

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$$R \sim^\tau S \Leftrightarrow \begin{cases} R \dot{\sim}^\tau S \\ \forall C[\cdot], C[O_R] \dot{\sim}^\tau C[O_S] \end{cases}$$

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Strong back-and-forth barbed congruence

$$R \sim^\tau S \Leftrightarrow \begin{cases} R \dot{\sim}^\tau S \\ \forall C[\cdot], C[O_R] \dot{\sim}^\tau C[O_S] \end{cases}$$

Example

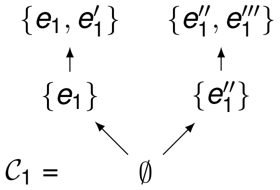
$$R = \langle i, b, a.P \rangle.\emptyset \rhd Q \qquad S = \langle j, a, b.Q \rangle.\emptyset \rhd P$$

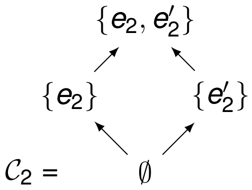
$O_R = O_S = \emptyset \rhd a.P + b.Q$, so $O_R \sim^\tau O_S$ but $R \dot{\nsim}^\tau S$!

Labelled configuration structure[4]

$\mathcal{C} = \langle E, C, \ell \rangle$ with $C \subset \mathcal{P}(E)$ and $\ell : E \rightarrow$ labels.

Example





$\ell_1(e_1) = \ell_1(e_1''') = a$

$\ell_1(e_1') = \ell_1(e_1'') = b$

$\ell_2(e_2) = a$

$\ell_2(e_2') = b$

[4] Winskel (1982)."Event structures for CCS"

Labelled configuration structure[4]

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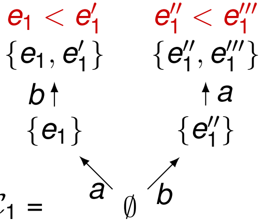
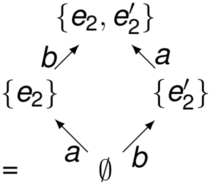
Example

$\mathcal{C}_1 =$



$\mathcal{C}_2 =$



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HHPB

$(\emptyset, \emptyset, \emptyset) \in \mathcal{R}$, and for $x_i \in C_i, e_i \in E_i, (x_1, x_2, f) \in \mathcal{R} \Rightarrow$

f label and order preserving bijection

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f label and order preserving bijection

$x_1 \xrightarrow{\alpha} x_1 \cup \{e_1\} \implies$
$x_2 \xrightarrow{\alpha} x_2 \cup \{e_2\}, (x_1 \cup \{e_1\}, x_2 \cup \{e_1\}, f') \in \mathcal{R}$

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- f label and order preserving bijection
- $x_1 \rightarrow^{\alpha} x_1 \cup \{e_1\} \implies$
  $x_2 \rightarrow^{\alpha} x_2 \cup \{e_2\}, (x_1 \cup \{e_1\}, x_2 \cup \{e_1\}, f') \in \mathcal{R}$
- $x_1 \rightsquigarrow^{\alpha} x_1 \setminus \{e_1\} \implies$
  $x_2 \rightsquigarrow^{\alpha} x_2 \setminus \{e_2\}, (x_1 \setminus \{e_1\}, x_2 \setminus \{e_1\}, f') \in \mathcal{R}$

Labelled configuration structure[4]

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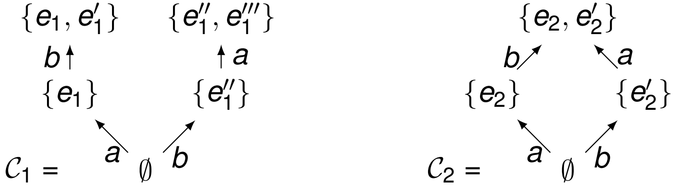
Example



$$\emptyset, \emptyset, \emptyset$$

$$\{e_1\}, \{e_2\}, e_1 \leftrightarrow e_2$$

$$\{e_1, e_1'\}, \{e_2, e_2'\}, e_1' \leftrightarrow e_2'$$

Reversible Configuration Structures

$$O_R = P \text{ such that } \emptyset \triangleright P \underbrace{\xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_i}}_{x_R=\{e_1,\ldots,e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

Reversible Configuration Structures

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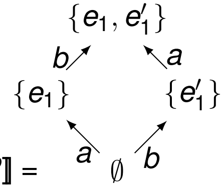
We can consider only forward transitions.

Example

$[\![P]\!] =$



$$P = \quad a.0 \mid \quad b.0$$

Reversible Configuration Structures

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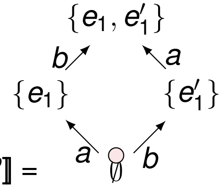
$[\![R]\!] =$ 

$$\emptyset \triangleright P \equiv \curlyvee.\emptyset \triangleright a.0 \mid \curlyvee.\emptyset \triangleright b.0$$

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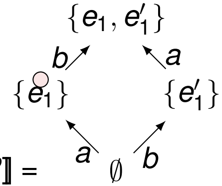

$$\emptyset \triangleright P \equiv \curlyvee.\emptyset \triangleright a.0 \mid \curlyvee.\emptyset \triangleright b.0$$

$$\xrightarrow{1:a} \langle 1, a\rangle.\curlyvee.\emptyset \triangleright 0 \mid \curlyvee.\emptyset \triangleright b.0$$

Reversible Configuration Structures

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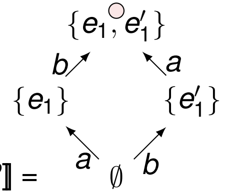
Example

$[\![R]\!] =$



$$\begin{aligned} \emptyset \triangleright P \equiv\ & \curlyvee .\emptyset \triangleright a.0 \mid \curlyvee .\emptyset \triangleright b.0 \\ & \xrightarrow{1:a} \langle 1, a \rangle . \curlyvee .\emptyset \triangleright 0 \mid \curlyvee .\emptyset \triangleright b.0 \\ & \xrightarrow{2:b} \langle 1, a \rangle . \curlyvee .\emptyset \triangleright 0 \mid \langle 2, b \rangle . \curlyvee .\emptyset \triangleright 0 \end{aligned}$$

Reversible Configuration Structures

$$O_R = P \text{ such that } \emptyset \triangleright P \underbrace{\xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_i}}_{x_R=\{e_1,\dots,e_i\}} R \qquad [\![R]\!] = ([\![O_R]\!], x_R)$$

We can consider only forward transitions.

Operational correspondence

let $[\![R]\!] = (\mathcal{C}, x)$:

$$R \xrightarrow{i:\alpha} S \Rightarrow (\mathcal{C}, x) \xrightarrow{\alpha} (\mathcal{C}, x \cup \{e\})$$

$$(\mathcal{C}, x) \xrightarrow{\alpha} (\mathcal{C}, x \cup \{e\}) \Rightarrow R \xrightarrow{i:\alpha} S$$

where $[\![S]\!] = (\mathcal{C}, x \cup \{e\})$, $\ell(e) = \alpha$ and the similarly for $\rightsquigarrow$.

RCCS | Configuration Structures

$R \sim^{\tau} S$ $\qquad$ $([\![O_R]\!], x_R)" \sim "([\![O_S]\!], x_S)$

$O_R \sim^{\tau} O_S \quad \xleftrightarrow{\;"="\;} \quad [\![O_R]\!] \dot{\sim} [\![O_S]\!]$

contextual characterisation of hhpb

$[\![O_R]\!] \dot{\sim} [\![O_S]\!] \Leftrightarrow$ for every context C, $[\![C[O_R]]\!] \dot{\sim} [\![C[O_S]]\!]$.

RCCS	Configuration Structures
$R \sim^{\tau} S$	$(\llbracket O_R \rrbracket, x_R)" \sim "(\llbracket O_S \rrbracket, x_S)$

$O_R \sim^{\tau} O_S \quad \xleftrightarrow{\text{" = "}} \quad \llbracket O_R \rrbracket \mathrel{\dot{\sim}} \llbracket O_S \rrbracket$

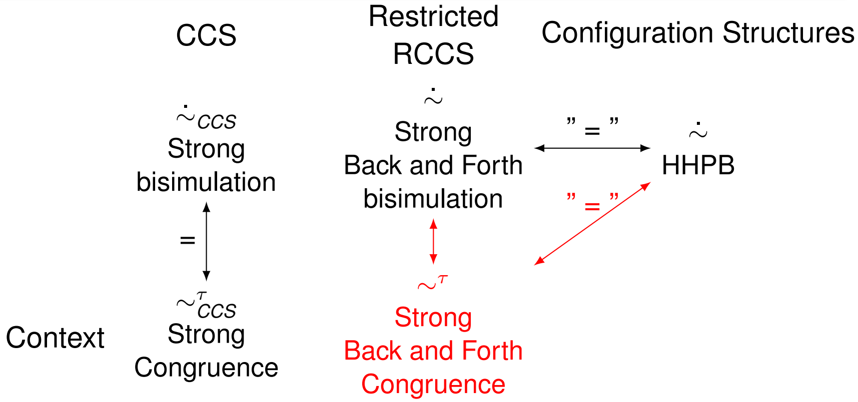
$O_R \sim^{\tau} O_S \quad \Longleftrightarrow \quad \llbracket O_R \rrbracket \sim^{\tau} \llbracket O_S \rrbracket$

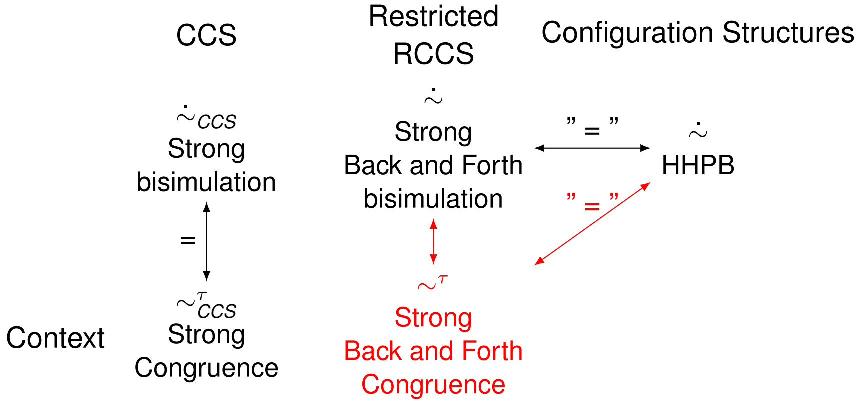
$\llbracket O_R \rrbracket \mathrel{\dot{\sim}} \llbracket O_S \rrbracket \quad \Updownarrow \quad \llbracket O_R \rrbracket \sim^{\tau} \llbracket O_S \rrbracket$

+ inductive approximations of hhpb

contextual characterisation of hhpb

$\llbracket O_R \rrbracket \mathrel{\dot{\sim}} \llbracket O_S \rrbracket \Leftrightarrow$ for every context C, $\llbracket C[O_R] \rrbracket \mathrel{\dot{\sim}} \llbracket C[O_S] \rrbracket$.


CCS
Restricted
RCCS
Configuration Structures
$\dot{\sim}_{CCS}$
Strong
bisimulation
=
$\sim^{\tau}_{CCS}$
Strong
Congruence
Context
$\dot{\sim}$
Strong
Back and Forth
bisimulation
" = "
$\dot{\sim}$
HHPB
" = "
$\sim^{\tau}$
Strong
Back and Forth
Congruence



Future work

- More general context for RCCS
- Weak case
- What to observe? directions?