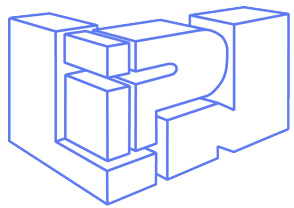
On pointers, graphs, and Logspace complexity.

9th project meeting of Complice

Clément Aubert



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1. L, graphs and tree
2. Pure Pointer Programs
3. Finite Automatons
4. SMM and KUM
5. Other models

The following problems are **L**-complete [JLM97]:

- Is Tree?
- Is Forest?
- Is Line?
- Acyclicity of undirected graph.
- Undirected s-t connectivity / Undirected reachability.

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The following problem is **NL**-complete:

- Directed s-t connectivity / Directed reachability

Presentation

- *PURPLE = PURe Pointer LanguagE ([HS08]),*
- *Only abstract pointers, no internal structure,* `forall`*-loops, [*`while`*-loops],*
- *Take as input a locally ordered graph.*

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Remark

If there is a total ordering on the nodes of the input graph, it is possible to encode any ***L*** *algorithm.*

Results

- *There is no PURPLE program that decides undirected reachability.*

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Links to other models of computation

- *Each* `forall-free` *program can be compiled into a JAG and vice versa.*
- *For each closed DTC*[a] *formula* ϕ *for locally ordered graphs there exists a program* M_ϕ *such that, for any finite locally ordered graph* Γ *of degree* d*,* $\Gamma \models \phi$ *iff* M_ϕ *recognizes* Γ*.*

[a] Deterministic Transitive Closure

Presentation

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- *States and transition function as usual,*
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Remark

$$\boldsymbol{L} = \cup_{k \geqslant 1} \mathcal{L}(2DFA(k))$$

$$\boldsymbol{NL} = \cup_{k \geqslant 1} \mathcal{L}(2NFA(k))$$

Results

Let $k \geqslant 1$

- *There are unary languages that shows* $\mathcal{L}(2DFA(k)) \subset \mathcal{L}(2DFA(k+1))$ *and the same holds for* $\mathcal{L}(2NFA(k))$.
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Links to other models of computation

- *The trade-off between (non-)deterministic two-way* $(k+1)$*-head finite automata and (non-)deterministic two-way* k*-head finite automata is non-recursive.*
- *Let* $k \geqslant 2$*, the trade-off between deterministic* $1DFA(k)$ *and non-deterministic pushdown automata is non-recursive.*

  "The gain in economy of description can be arbitrary."
  [HKM09]

Presentation (Storage Modification Machines [Sch80])

- *Data structure processed:* $\Delta = (X, a, p)$ *where*
  - X *is a finite set of nodes,*
  - $a \in X$ *is the center (the active node),*
  - $p = (p_\alpha \mid \alpha \in X)$ *is the family of pointer mappings* $p_\alpha : X \to X$

  $p_\alpha(x) = y$ *means that the pointer with label* α *originating from* x *goes to* y*. We define* $p^* : \Delta^* \to X$ *recursively:*

$$p^*(\square) = a, \text{ were } \square \text{ is the empty word}$$

$$p^*(W\alpha) = p_\alpha(p^*(W)) \text{ for all } \alpha \in \Delta, W \in \Delta^*$$

- *Finite control given as a program made of labels (as in ALGOL) and instructions (such as input, output, goto, new, set, if).*

Presentation (Kolmogorov Uspenskii Machines [KU58])

Exactly like the SMM, except that it operates on undirected graph.

Look at the board!

Look at the board!

In space n, with k symbols, a Turing machine can encode

$$O(n \log k)$$

bits of information, a SMM can encode

$$O(n\, k \log n)$$

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- *Takes a d-maze as input,*
- *Deterministic control,*
- *Finite set of pebbles.*

The storage of a JAG with P pebbles and Q states is $P \log N + \log Q$.

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Not to mention other pebbles automaton, Knuth's Linking automaton...


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