

Ψ^2 a Software Tool for the Perfect Simulation of Finite Queuing Networks

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1 Introduction

Markovian networks of finite capacity queues are widely used models for performance evaluation of systems and networks. Unfortunately, excepted in some specific situations, these models are not tractable analytically [2]. Approximation techniques, aggregation, fluidification have been proposed to capture the behavior such systems. But, in most complex cases, simulation remains the only tool to estimate the steady state of the system.

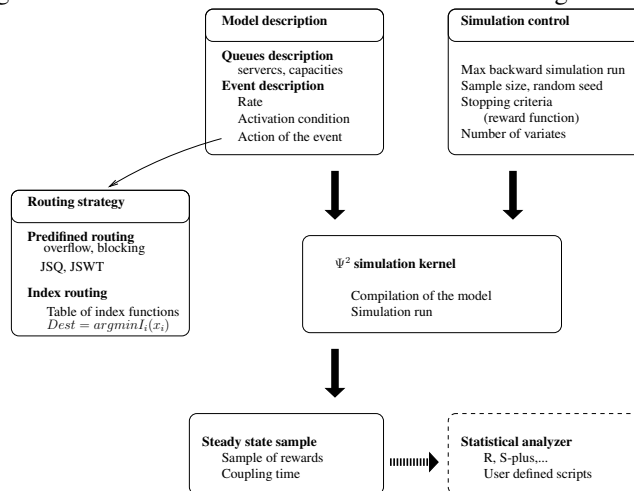
Classical simulation iterates from an initial state and estimates the steady-state on a long run trajectory via the ergodic theorem. The first problem of this direct simulation is the simulation control of the burn-in time period that ensures that the process have reached the steady state. The second difficulty is related to the auto-correlation of a one trajectory sample. Approximations or regenerative arguments should be used to compute confidence intervals. For large state-space systems regeneration arguments fail and should be adapted with a high knowledge on the system.

Perfect simulation provides a new technique to sample steady-state and avoids the burn-in time period. When the simulation algorithm stops, the returned state value is in steady-state. Initiated by Propp and Wilson [5] in the context of statistical physics, this technique is based on a coupling from the past scheme that, provided some conditions on the system, ensures convergence in a finite time to steady-state. This approach have been successfully applied in various domains, stochastic geometry, interacting particle systems, statistical physics, networking [1, 4], etc.

We applied this technique first to Markov chain with sparse transition matrix [8, 7], and to queueing networks with finite capacities and complex routing strategies [9]. The software Ψ^2 have been developed to validate this simulation approach and applied to in the context of low probability events estimation [10, 6]. The aim of this note is to present the features of this free software which is distributed at <http://psi.gforge.inria.fr>. The de-

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sign of the software architecture is shown in the next figure.



2 Model description

The model is based on a multiple servers queueing network description. The number of servers and the capacities of queues are fixed. All capacities are finite so that a state of the network will be modelled by a vector in a product of integer intervals.

An event e is considered as a parameter of a global transition function $\Phi(x, e)$ on the state space. Each event is driven by a stochastic Poisson process. The underlying discrete time Markov chain is obtained by uniformization (GSMP model). A triggered event e that cannot be activated on a state x induces a skip operation ie $\Phi(x, e) = x$.

A library contains a list of pre-defined events. The arguments of each event is an “origin” queue and a list of destination queues (DQ) with the implicit priority in the list.

type	action
1	Server departure
2	External arrival to the first empty room in the list DQ
3	Multi-server departure to DQ
4	Join the shortest queue in DQ
⋮	⋮

Moreover, there is the possibility to implement a user-defined

routing strategy based on index policy : for a given index routing event e the destination queue is computed as the $\text{argmin}_i I_i(x_i)$, the specific index functions I_i are stored in separate files.

Such networks do not have a product form because of overflow, blocking or state dependent routing. But all of these events preserve monotonicity on the state space with the component-wise ordering. Consequently the backward coupling is highly improved by drawing only two trajectories from the minimal state m (all queues are empty) and the maximal one M (all queues are full).

Simulation kernel

```

n ← 1; E[1] ← Generate-event()
repeat
  n ← 2n; y(M) ← M; y(m) ← m;
  for i = n downto 1 do
    if n > n/2 then
      E[i] ← Generate-event() {generate event -i according
        to uniformized distribution of events}
    end if
    y(M) ← Φ(y(M), E[i]); y(m) ← Φ(y(m), E[i]);
    {apply the transition given by event E[i]}
  end for
until All y(x) are equal
return y(x)

```

3 Simulation parameters, control and output analysis

The backward coupling scheme needs to store the sequence of events E . The simulation could take an arbitrary long time to couple, but it has been shown that the mean memory size needed for a trajectory a feed-forward network is quadratic in the capacities of queues and linear in the ratio of the global rate of the system and the rate of each queue [3].

In some cases, when we need only a monotonic reward on steady-state, the stopping condition is strengthened by $\text{rewards}(y(M)) = \text{reward}(y(m))$. This reward function is given to the simulator as an external C encoded function. This could heavily reduce the computation time of expected reward. By brute force and reward function, one could estimate low probabilities sets of states [6], up to 10^{-6} .

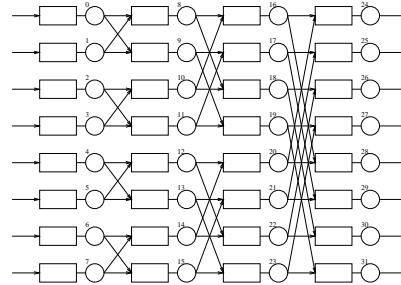
To improve the efficiency of the simulation, we also implement variance reduction techniques by driving parallel antithetic trajectories. The simulation time could then be reduced by a factor 2 or 3 [10].

After running the simulation kernel, we get a sample of independent rewards in steady-state. The independence property is guaranteed by the fact that we use independent trajectories. Consequently, we could use all classical theorems from statistics to estimate steady-state parameters.

We could use for example the R software, our own scripts or other statistical tools.

4 Applications

Consider a simple multistage switching network, with a capacity of queues 99 and a uniform routing scheme. The input rate is 0.9 and the rate of each queue 1.



The size of the state-space is 10^{64} . The performance index under study is the probability that the first queue on the last level is over 50. Such heavy loaded models are usually untractable with classical methods. On a Pentium PIV, 3GHz, 2Gb memory the performances are 106ms per generated state and only 75ms to generate an independent sample of the performance index. In this case we estimate a very low probability event $\mathbb{P}(N > 50) \simeq 4.5 \cdot 10^{-3} \pm 2 \cdot 10^{-4}$, with a 95% confidence level, in few hours.

This software have been successfully applied in many situations such as grid scheduling, call center dimensionning, optimal routing etc.

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