Independence of the MIN principle from the PHP principle over bounded arithmetic

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Abstract

The minimization principle MIN($\prec$) studied in bounded arithmetic says that a strict linear ordering $\prec$ on any finite interval $[0, \dots, n)$ has the minimal element. We shall prove that bounded arithmetic theory $T^1_2(\prec)$ augmented by instances of the pigeonhole principle for all $\Delta^b_1(\prec)$ formulas does not prove MIN($\prec$).

As a demonstration of our method, we derive similar independence for another prominent Σ^b_2-principle TOUR(G) from the aforementioned pigeonhole principle, where TOUR(G) states that, given an orientation G of the K_n, there is a subset $S \subseteq K_n$ of size $\leq \log n$ with the property that any vertex from $K_n \setminus S$ has directed edge towards some element of S.

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